A) Compute the (sine/cosine) trigonometric Fourier series of the periodic signal shown in Figure P3A. [14 points]

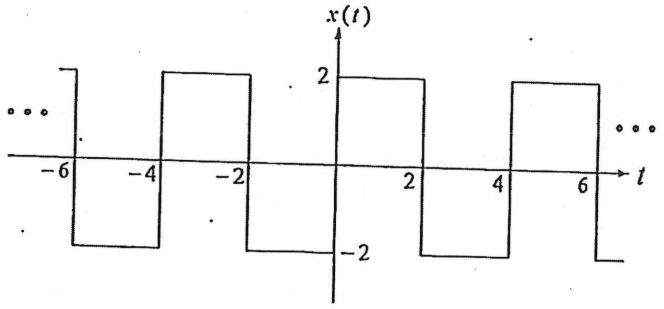


Figure P3A

This is an odd function $\Rightarrow a_0 = 0$ & $a_k = 0$

$T = 4 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

$$b_k = \frac{4}{T}\int_0^{T/2} x(t)\sin(k\omega_0 t)\,dt$$

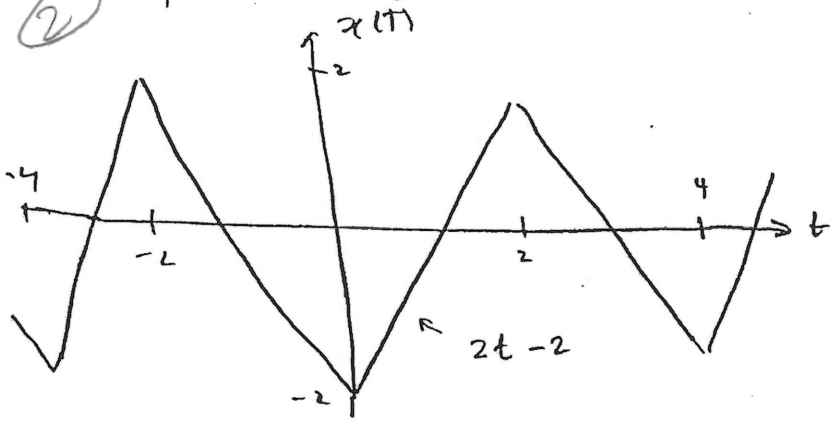
$$= 2\int_0^2 \sin\left(k\tfrac{\pi}{2}t\right)dt = \frac{-4}{k\pi}\left[\cos\left(k\tfrac{\pi}{2}t\right)\right]_0^2$$

$$= \frac{-4}{k\pi}\left[\cos(k\pi) - 1\right] = \frac{4}{k\pi}\left[1 - \cos(k\pi)\right]$$

$$\therefore b_k = \begin{cases} \frac{8}{k\pi}, & k \text{ odd} \\ 0, & k \text{ even} \end{cases}$$

$$\therefore x(t) = \sum_{\substack{k=1 \\ k\,\text{odd}}}^{\infty} \frac{8}{k\pi}\sin\left(k\tfrac{\pi}{2}t\right), \quad -\infty < t < \infty$$

(2) Compute the trigonometric Fourier transform



$\Rightarrow$ Even function

$\Rightarrow b_k = 0$

$T = 4 \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$a_0 = \frac{2}{T}\int_0^{T/2} x(t)\,dt = \frac{2}{4}\int_0^2 (2t-2)\,dt = \frac{2}{4}\left[t^2 - 2t\right]_0^2 = 0$$

$$a_k = \frac{4}{T}\int_0^{T/2} x(t)\cos(k\omega_0 t)\,dt = \frac{4}{4}\int_0^2 (2t-2)\cos\left(kt\tfrac{\pi}{2}\right)dt$$

let $u = 2t-2$ and $dv = \cos\left(kt\frac{\pi}{2}\right)dt$

$\Rightarrow du = 2dt$ and $v = \frac{2}{k\pi}\sin\left(kt\frac{\pi}{2}\right)$

$$\therefore \int_0^2 u\,dv = uv\Big|_0^2 - \int_0^2 v\,du$$

$$\therefore a_k = \left[(2t-2)\left(\frac{2}{k\pi}\sin\left(kt\tfrac{\pi}{2}\right)\right)\right]_0^2 - \int_0^2 \frac{4}{k\pi}\sin\left(kt\tfrac{\pi}{2}\right)dt$$

$$= \left[(4-2)\left(\frac{2}{k\pi}\sin(k\pi) - 0\right)\right] + \frac{4}{k\pi}\cdot\frac{2}{k\pi}\left[\cos\left(kt\tfrac{\pi}{2}\right)\right]_0^2$$

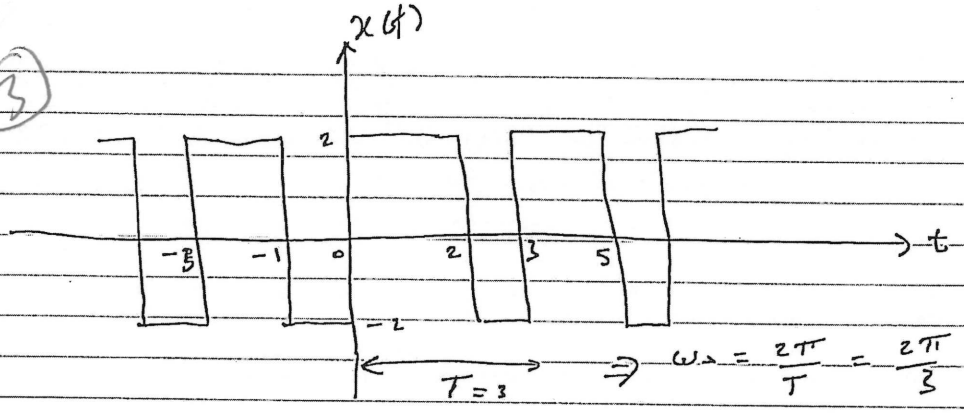
$$= \left[\frac{4}{k\pi}\sin(k\pi)\right] + \frac{8}{(k\pi)^2}\left[\cos(k\pi) - 1\right]$$

$\sin(k\pi) = 0$, for $k = 1, 2, 3, \ldots$

$$a_k = \frac{8}{(k\pi)^2}\left[\cos(k\pi) - 1\right] \quad \text{or} \quad a_k = \begin{cases} 0, & k \text{ is even} \\ \frac{-16}{\pi^2 k^2}, & k \text{ is odd} \end{cases}$$

$$\Rightarrow x(t) = \sum_{\substack{k=1 \\ k \text{ odd}}}^{\infty} \frac{16}{\pi^2 k^2}\cos(k\omega_0 t), \quad \text{and} \quad \omega_0 = \frac{\pi}{2}$$

(3)



$$\Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{3}\int_0^3 x(t)\,dt = \frac{1}{3}\left[\int_0^2 2\,dt + \int_2^3 (-2)\,dt\right] = \frac{1}{3}\left[4 - 2\right] = \frac{2}{3}$$

$$a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt = \frac{2}{3}\int_0^2 2\cos\left(\frac{2\pi k}{3}t\right)dt + \frac{2}{3}\int_2^3 (-2)\cos\left(\frac{2\pi k}{3}t\right)dt$$

$$= \frac{2}{3}\left[2\left(\frac{3}{2\pi k}\right)\sin\left(\frac{2\pi k}{3}t\right)\right]_0^2 + \frac{2}{3}\left[(-2)\left(\frac{3}{2\pi k}\right)\sin\left(\frac{2\pi k}{3}t\right)\right]_2^3$$

$$= \frac{2}{\pi k}\left[\sin\left(\frac{4\pi}{3}k\right) - \sin(2\pi k) + \sin\left(\frac{4\pi}{3}k\right)\right]$$

$$= \frac{2}{\pi k}\left[2\sin\left(\frac{4\pi k}{3}\right) - \sin(2\pi k)\right]$$

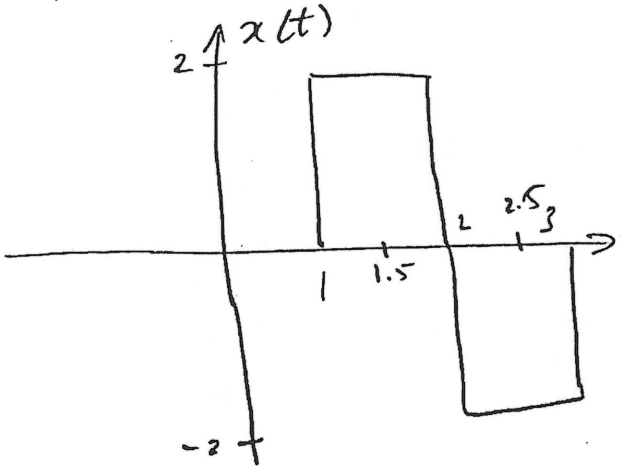
$\sin(2\pi k) = 0$

$\sin\left(\frac{4\pi k}{3}\right) = 0$ for $k = 0, 3, 6, 9, \ldots$

$$\Rightarrow a_k = \frac{4}{\pi k}\sin\left(\frac{4\pi k}{3}\right) \text{ for } k = 1, 2, 4, 5, 7, 8, 10, 11, \ldots$$

(4) Compute the Fourier transform of the following signal

4



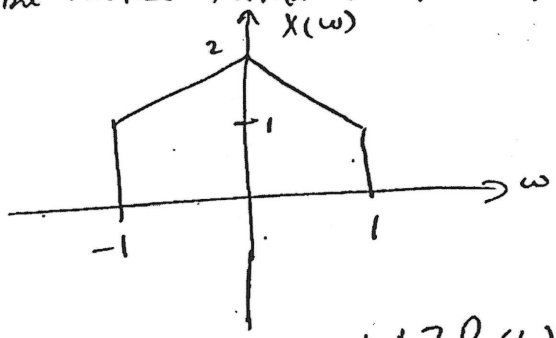
$\Rightarrow x(t) = P_1(t-1.5) - P_1(t-2.5)$

$P_\tau \longleftrightarrow \tau\, \text{sinc}\left(\frac{\tau\omega}{2\pi}\right)$

$P_\tau(t-c) \longleftrightarrow \tau\, \text{sinc}\left(\frac{\tau\omega}{2\pi}\right) e^{-j\omega c}$

$\therefore \quad x(t) \longleftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right) e^{-j\frac{3\omega}{2}} - \text{sinc}\left(\frac{\omega}{2\pi}\right) e^{-j5\omega/2}$

(5) Compute the inverse Fourier transform of the frequency function show below



$X(\omega) = P_2(\omega) + \left[1 - \frac{2|\omega|}{2}\right] P_2(\omega)$

$P_2(t) \longleftrightarrow 2\,\text{sinc}\left(\frac{2\omega}{2\pi}\right)$ ← even function

$\therefore \quad 2\,\text{sinc}\left(\frac{2t}{2\pi}\right) \longleftrightarrow 2\pi P_2(-\omega) = 2\pi P_2(\omega)$

$\therefore \quad P_2(\omega) \longleftrightarrow \frac{1}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right)$

$$\left(1-\frac{2|t|}{\tau}\right)P_\tau \longleftrightarrow \frac{\tau}{2}\,\text{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$$

5

$$\therefore \frac{\tau}{2}\,\text{sinc}^2\left(\frac{\tau t}{4\pi}\right) \longleftrightarrow 2\pi\left(1-\frac{2|-\omega|}{\tau}\right)P_\tau(-\omega)$$

$$\longleftrightarrow 2\pi\left(1-\frac{2|\omega|}{\tau}\right)P_\tau(\omega)$$

$$\therefore \left[1-\frac{2|t|}{2}\right]P_2(\omega) \longleftrightarrow \frac{2}{2}\cdot\frac{1}{2\pi}\,\text{sinc}^2\left(\frac{2t}{4\pi}\right)$$

$$\therefore [1-|t|]\,P_2(\omega) \longleftrightarrow \frac{1}{2\pi}\,\text{sinc}^2\left(\frac{t}{2\pi}\right)$$

$$\therefore x(t) = \frac{1}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right) + \frac{1}{2\pi}\,\text{sinc}^2\left(\frac{t}{2\pi}\right),\ -\infty < t < \infty$$

6

A signal has Fourier transform as

$$Y(\omega) = \frac{1}{j}\left[2\,\text{sinc}\left(\frac{2\omega}{\pi}-\frac{1}{4}\right) - 2\,\text{sinc}\left(\frac{2\omega}{\pi}+\frac{1}{4}\right)\right)$$

$$X(\omega) = 2j\left[\text{sinc}\left(\frac{2\omega}{\pi}+\frac{1}{4}\right) - \text{sinc}\left(\frac{2\omega}{\pi}-\frac{1}{4}\right)\right]$$

$$x(t)\sin(\omega_0 t) \longleftrightarrow \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$$

$$P_\tau(t) \longleftrightarrow \tau\,\text{sinc}\left(\frac{\tau\omega}{2\pi}\right)$$

$$P_4(t) \longleftrightarrow 4\,\text{sinc}\left(\frac{2\omega}{\pi}\right)$$

$$\therefore P_4(t)\sin\left(\frac{1}{4}t\right) \longleftrightarrow j\left[4\,\text{sinc}\left(\frac{2\omega}{\pi}+\frac{1}{4}\right) - 4\,\text{sinc}\left(\frac{2\omega}{\pi}-\frac{1}{4}\right)\right]$$

$$\longleftrightarrow 4j\left[\text{sinc}\left(\frac{2\omega}{\pi}+\frac{1}{4}\right) - \text{sinc}\left(\frac{2\omega}{\pi}-\frac{1}{4}\right)\right]$$

$$x(t) = \frac{1}{2}P_4(t)\sin\left(\frac{1}{4}t\right).$$

(7) $X(\omega) = \frac{1}{j}\left[2\,\text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right) - 2\,\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right)\right]$ ←

(6)

Find $x(t)$

Table 3.1

Property: $x(t)\sin(\omega_0 t) \leftrightarrow \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$

$$X(\omega) = \frac{-2}{j}\left[\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right) - \text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right)\right]$$

$$= 2j\left[\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right) - \text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right)\right]$$

$$= 4\left(\frac{j}{2}\left[\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right) - \text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right)\right]\right)$$

$\therefore \boxed{\omega_0 = \frac{1}{4}}$ $\qquad P_4(t) \leftrightarrow 4\,\text{sinc}\left(\frac{2\omega}{\pi}\right)$

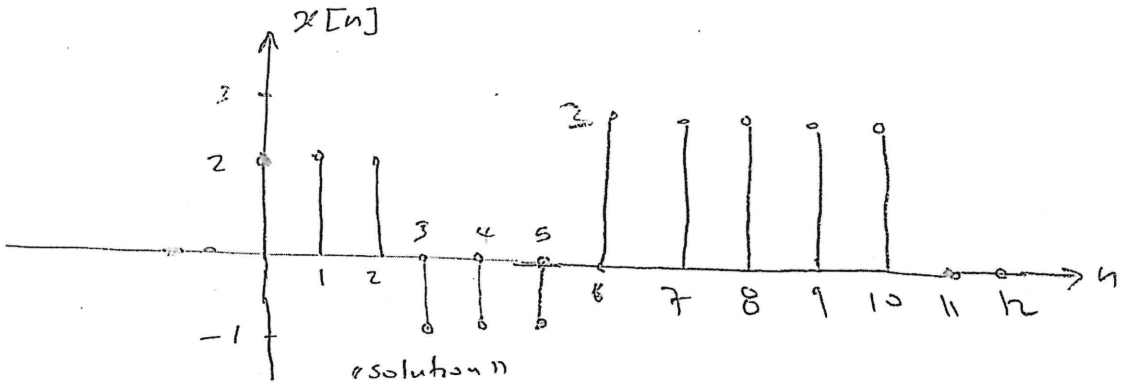
$\therefore \boxed{x(t) = P_4(t)\sin\left(\frac{1}{4}t\right)}$

$$X(\omega) = \frac{j}{2}\left[4\,\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right) - 4\,\text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right)\right]$$

$$X(\omega) = 2j\left[\text{sinc}\left(\frac{2\omega}{\pi} + \frac{1}{4}\right) - \text{sinc}\left(\frac{2\omega}{\pi} - \frac{1}{4}\right)\right] \quad \checkmark$$

(8) Compute the DTFT of the following D-T signal

(7)



esolution

$$\therefore x[n] = 2P_2[n-1] - P_2[n-4] + 3P_4[n-8]$$

from Table 4.1, $P_{2q}[n] \longleftrightarrow \dfrac{\sin[(q+\frac{1}{2})\Omega]}{\sin(\Omega/2)}$

where q is one half of pulse time ($q = L/2$)

$$\therefore 2P_2[n] \longleftrightarrow 2\,\frac{\sin[1.5\Omega]}{\sin[\Omega/2]}$$

from table 4.2, $x[n-q] \longleftrightarrow X(\Omega)e^{-jq\Omega}$

$$\therefore 2P_2[n-1] \longleftrightarrow 2\,\frac{\sin[1.5\Omega]}{\sin[\Omega/2]}e^{-j\Omega}$$

Then

$$-P_2[n-4] \longleftrightarrow -\frac{\sin[1.5\Omega]}{\sin[\Omega/2]}e^{-4j\Omega}$$

and

$$3P_4[n-8] \longleftrightarrow 3\,\frac{\sin[2.5\Omega]}{\sin[\Omega/2]}e^{-j8\Omega}$$

$$\therefore X(\Omega) = \frac{1}{\sin(\Omega/2)}\left[2\sin[1.5\Omega]e^{-j\Omega} - \sin[1.5\Omega]e^{-j4\Omega} + 3\sin[2.5\Omega]e^{-j8\Omega}\right.$$

9. Compute the DTFT of the following DT signals

8

(a) $x[n] = (-0.5)^n u[n]$

From table 4.1, $a^n u[n] \leftrightarrow \frac{1}{1 - a e^{-j\Omega}}$, $|a|<1$

$$X(\Omega) = \frac{1}{1+0.5e^{-j\Omega}} = \frac{e^{j\Omega}}{e^{j\Omega}+0.5}$$

(b) $x[n] = n(0.5)^n u[n]$, Let $g[n]=(0.5)^n u[n] \Rightarrow G(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega}-0.5}$

$\because x[n] = n g[n]$

$$\Rightarrow X(\Omega) = j\frac{d}{d\Omega} G(\Omega) = j \frac{d}{d\Omega}\left(\frac{e^{j\Omega}}{e^{j\Omega}-0.5}\right)$$

$$= j\frac{(e^{j\Omega}-0.5)(j e^{j\Omega}) - e^{j\Omega}(j e^{j\Omega})}{(e^{j\Omega}-0.5)^2}$$

$$= \frac{-e^{j2\Omega} + 0.5 e^{j\Omega} + e^{-j2\Omega}}{(e^{j\Omega}-0.5)^2} = \frac{0.5 e^{j\Omega}}{(e^{j\Omega}-0.5)^2}$$

(c)

$x[n] = 10(0.8)^n \cos(2n) u[n]$

From (a)

$$10(0.8)^n u[n] \leftrightarrow 10 \frac{e^{j\Omega}}{e^{j\Omega}-0.8}$$

Use multiplication by $\cos(2n)$ property (table 4.2)

$$\therefore \left(10(0.8)^n u[n]\right)\cos(2n) \leftrightarrow \frac{10}{2}\left[\frac{e^{j(\Omega+2)}}{e^{j(\Omega+2)}-0.8} + \frac{e^{j(\Omega-2)}}{e^{j(\Omega-2)}-0.8}\right]$$

(10) A discrete-time signal $x[n]$ has DTFT

$$X(\Omega) = \frac{1}{e^{j\Omega} + b}$$

(9)

Determine the DTFT $V(\Omega)$ of the following signals

(a) $V[n] = x[n] * x[n]$

$$V(\Omega) = X(\Omega)\,X(\Omega) = \frac{1}{(e^{j\Omega} + b)^2}$$

(b) $V[n] = x[-n] * x[n]$

$$x[-n] \longleftrightarrow X(-\Omega)$$

$$V(\Omega) = X(-\Omega)\,X(\Omega) = \left(\frac{1}{e^{-j\Omega} + b}\right)\left(\frac{1}{e^{j\Omega} + b}\right)$$

$$= \frac{1}{1 + b e^{j\Omega} + b e^{-j\Omega} + b^2}$$

(c) $V[n] = x[n] e^{j2n}$

Using table 4.2 $\quad x[n] e^{j n \Omega_0} \longleftrightarrow X(e^{-\Omega_0})$

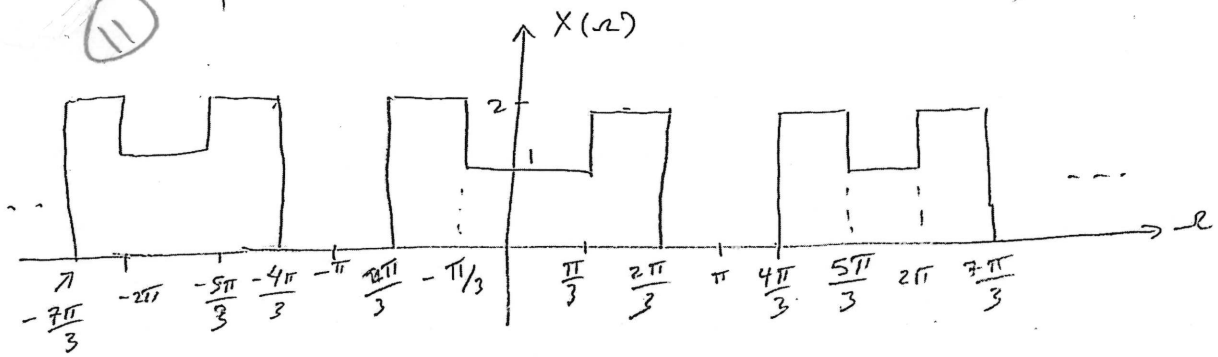
$$\therefore V(\Omega) = \frac{1}{(e^{j(\Omega - 2)} + b)}$$

(d) $V[n] = x[n] - 2x[n-2]$

$$x[n-q] \longleftrightarrow X(\Omega)\, e^{-jq\Omega}$$

$$\therefore V(\Omega) = \frac{1}{e^{j\Omega} + b} - 2\left(\frac{1}{e^{j\Omega} + b}\right) e^{-j2\Omega} = \frac{1 - 2e^{-j2\Omega}}{e^{j\Omega} + b}$$

11 Compute the inverse DTFT of the following function



10

For $-\pi \le \Omega \le \pi$, $X(\Omega) = 2 P_{\frac{4\pi}{3}}(\Omega) - P_{\frac{2\pi}{3}}(\Omega)$

or $X(\Omega) P_{2\pi} = 2 P_{\frac{4\pi}{3}}(\Omega) - P_{\frac{2\pi}{3}}(\Omega)$

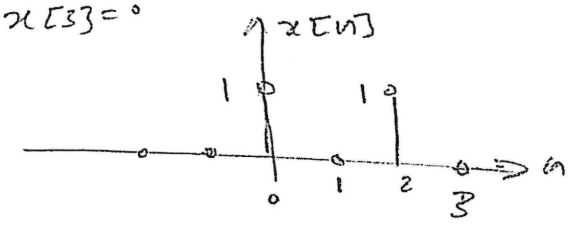
$$\therefore x[n] = 2\left(\frac{4\pi/3}{2\pi} \operatorname{sinc}\left[\frac{(4\pi/3)n}{2\pi}\right]\right) - \frac{2\pi/3}{2\pi} \operatorname{sinc}\left[\frac{(2\pi/3)n}{2\pi}\right]$$

$$= \frac{4}{3} \operatorname{sinc}\left(\frac{2n}{3}\right) - \frac{1}{3} \operatorname{sinc}\left(\frac{n}{3}\right)$$

12 Compute the rectangular form of the four point DFT of the following signals

(a) $x[0] = 1$ and $x[2] = 1$, and $x[n] = 0$ for other n

$x[1] = 0$, $x[3] = 0$



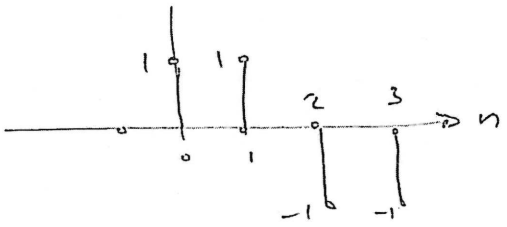
$$X_k = \sum_{n=0}^{N-1} x[n] e^{-j 2\pi k n / N}$$

$N = 4$ $\quad \therefore X_k = x[0] e^{0} + 0 + x[2] e^{-j\frac{2\pi k(2)}{4}} + 0$

$$= 1 + e^{-j\pi k} = 1 + \cos(\pi k), \quad \sin(\pi k) = 0$$

$X_0 = 2$, $X_1 = 0$, $X_2 = 2$, $X_3 = 0$

(b) $x[0]=1,\ x[1]=1,\ x[2]=-1,\ x[3]=-1,\ x[n]=0$ for other n



$N=4$

$$X_k = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi kn}{N}}$$

$$X_k = e^{0} + e^{-j\frac{2\pi k}{4}} - e^{-j\frac{4\pi k}{4}} - e^{-j\frac{6\pi k}{4}}$$

$$= 1 + e^{-j\frac{\pi}{2}k} - e^{-j\pi k} - e^{-j\frac{3}{2}\pi k}$$

$e^{-j\pi n} = \cos(\pi k)$, because $\sin(\pi k)=0$

$e^{-j\frac{\pi}{2}k} = \cos(\frac{\pi}{2}k) - j\sin(\frac{\pi}{2}k)$

$e^{-j\frac{3\pi}{2}k} = \cos(\frac{3\pi}{2}k) - j\sin(\frac{3\pi}{2}k)$

$= \cos(\frac{\pi}{2}k) + j\sin(\frac{\pi}{2}k)$

$\therefore X_k = 1 - \cos(\pi k) - j2\sin(\frac{\pi}{2}k)$

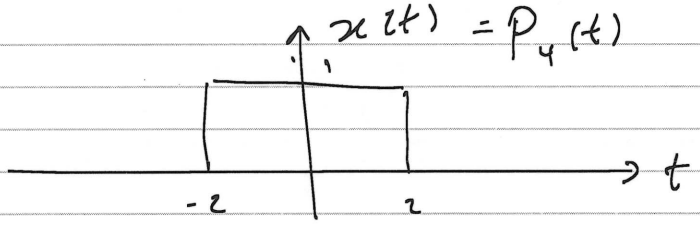
$X_0 = 0,\ X_1 = 2 - j2,\ X_2 = 0,\ X_3 = 2 + j2$

(c) $x[0]=1,\ x[1]=0,\ x[2]=-1,\ x[3]=0,\ x[n]=0$ for other n

$X_k = e^{0} + 0 - e^{-j\pi k} + 0 = 1 - \cos(\pi k)$, $n = 0, 1, 2, 3$

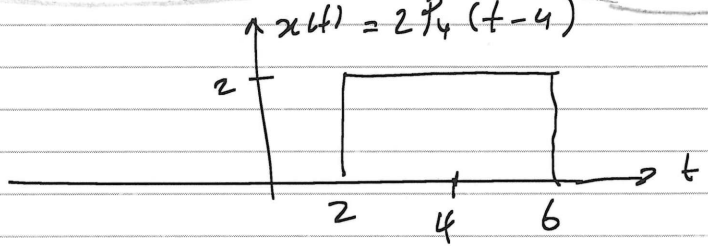
$\therefore X_0 = 0,\ X_1 = 2,\ X_2 = 0,\ X_3 = 2$

Ex 13



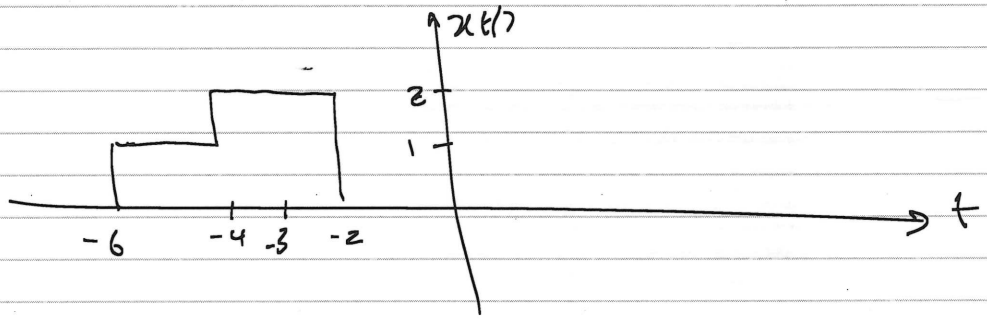
$$P_4(t) \longleftrightarrow 4\,\text{sinc}\left(\frac{4\omega}{2\pi}\right)$$

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$$2P_4(t-4) \longleftrightarrow (2)(4)\,\text{sinc}\left(\frac{4\omega}{2\pi}\right) e^{-j4\omega}$$

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(a)

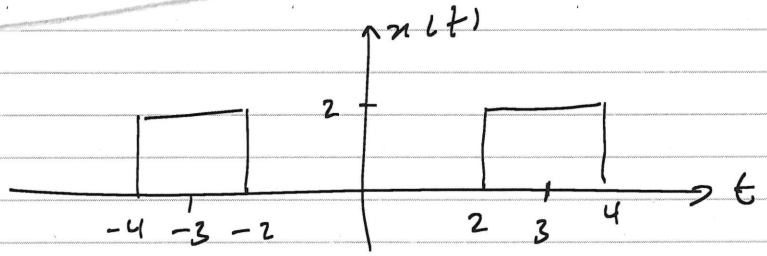
$$\therefore x(t) = P_4(t+4) + P_2(t+3)$$

$$\therefore X(\omega) = 4\,\text{sinc}\left(\frac{4\omega}{2\pi}\right) e^{j4\omega} + 2\,\text{sinc}\left(\frac{2\omega}{2\pi}\right) e^{j3\omega}$$

OR (b) $x(t) = P_2(t+5) + 2P_2(t+3)$

$$X(\omega) = 2\,\text{sinc}\left(\frac{2\omega}{2\pi}\right) e^{j5\omega} + 4\,\text{sinc}\left(\frac{2\omega}{2\pi}\right) e^{j3\omega}$$

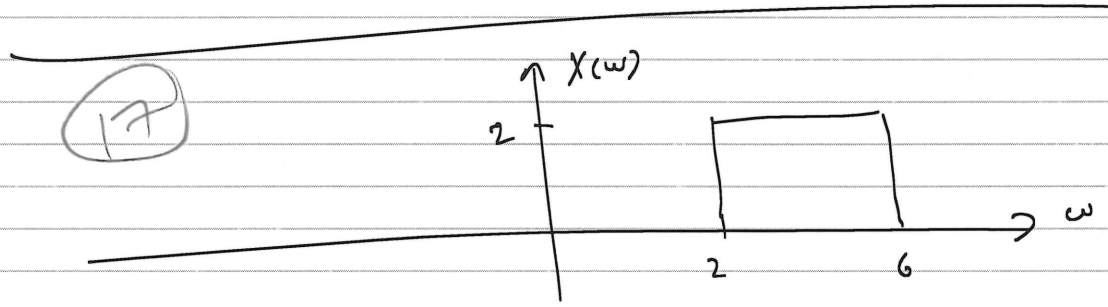
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$$x(t) = 2P_2(t+3) + 2P_2(t-3)$$

$$X(\omega) = 4\,\text{sinc}\left(\frac{2\omega}{2\pi}\right)e^{j3\omega} + 4\,\text{sinc}\left(\frac{2\omega}{2\pi}\right)e^{-j3\omega}$$

OR (b) use $x(t) = x(t)\cos(\omega t)$



$x(t) = ?$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(t) \longleftrightarrow 2\pi\, x(-\omega)$$

$$X(\omega) = 2P_4(\omega - 4)$$

$$P_\tau(t-c) \longleftrightarrow \tau\,\text{sinc}\left(\frac{\tau\omega}{2\pi}\right)e^{-j\omega c}$$

$$2P_4(t-4) \longleftrightarrow 8\,\text{sinc}\left(\frac{4\omega}{2\pi}\right)e^{-j4\omega}$$

$$8\,\text{sinc}\left(\frac{4t}{2\pi}\right)e^{-j4t} \longleftrightarrow (2\pi)\left(2P_4(-\omega-4)\right)$$

$$\frac{8}{2\pi}\,\text{sinc}\left(\frac{4t}{2\pi}\right)e^{-j4t} \longleftrightarrow 2\underbrace{P_4(-\omega-4)}_{=P_4(\omega-4)}$$

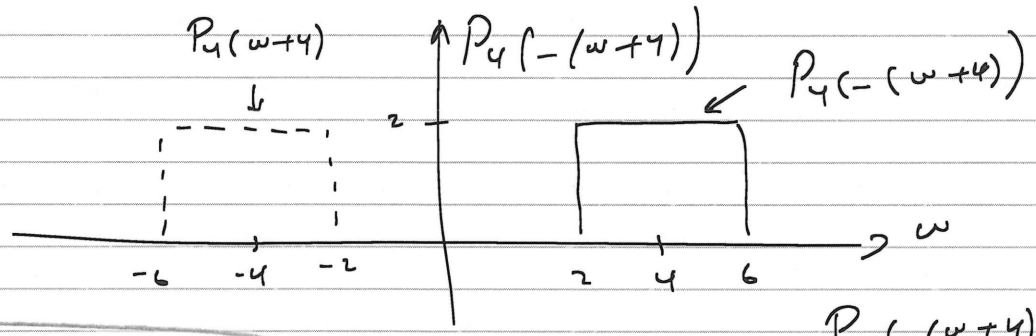
$$\therefore \frac{4}{\pi}\,\text{sinc}\left(\frac{2t}{\pi}\right)e^{-j4t} \longleftrightarrow 2P_4(\omega-4)$$

$$\therefore x(t) = \frac{4}{\pi}\,\text{sinc}\left(\frac{2t}{\pi}\right)e^{-j4t}$$

$P_4(-\omega-4) = P_4(-(\omega+4))$

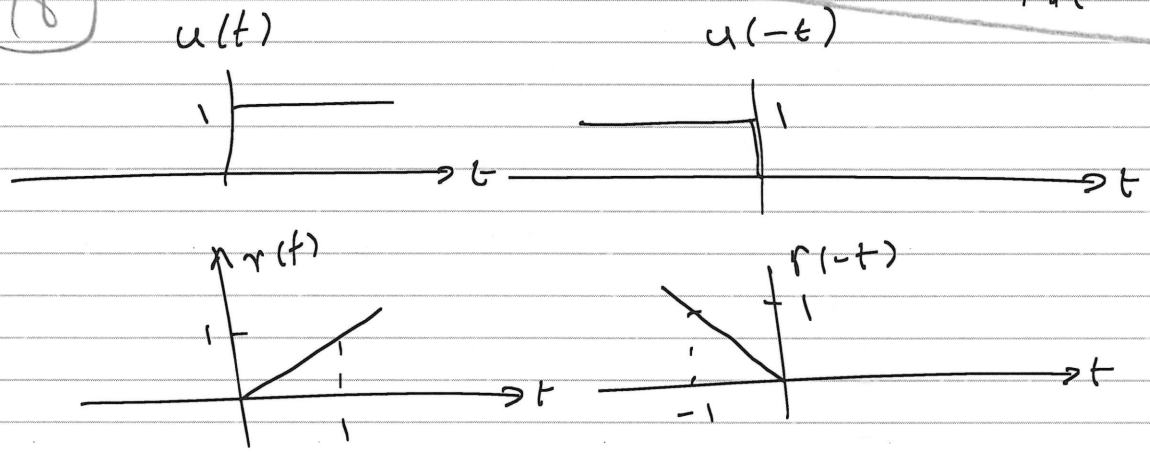
$P_\tau(\omega)$ is Even

But $P_\tau(\omega-c)$ is not Even



$\therefore P_4(-(\omega+4)) = P_4(-\omega-4) = P_4(\omega-4)$

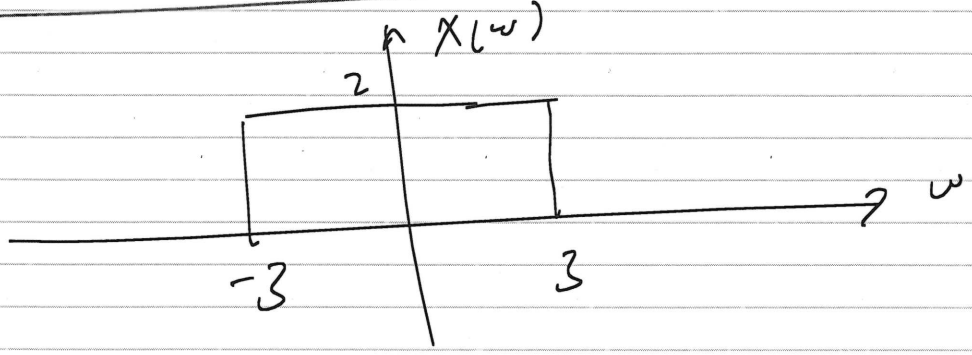
18



$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t\geq 0 \end{cases}$$

$$u(-t) = \begin{cases} 0, & -t<0 \\ 1, & -t\geq 0 \end{cases} = \begin{cases} 0, & t>0 \\ 1, & t\leq 0 \end{cases}$$

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$x(t) = ?$

$X(\omega) = 2\,P_6(\omega)$

$$2P_6(t) \leftrightarrow 12\,\text{sinc}\left(\frac{6\omega}{2\pi}\right)$$

C ✕ S

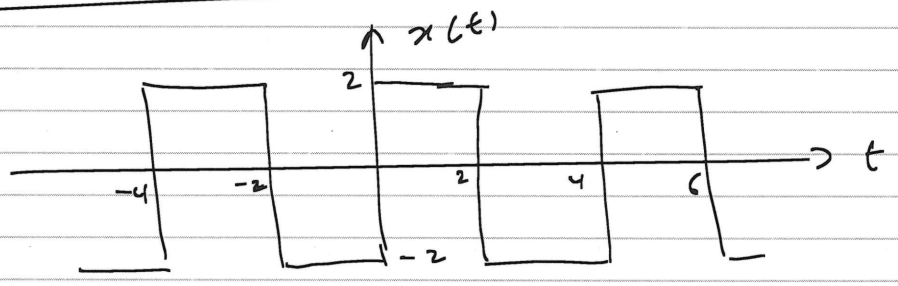
$$12\,\text{sinc}\left(\frac{6t}{2\pi}\right) \leftrightarrow (2\pi)(2P_6(-\omega)) \qquad , P_\tau(-\omega) = P_\tau(\omega) \text{ even}$$

$$\leftrightarrow (2\pi)(2P_6(\omega))$$

$$\therefore \frac{12}{2\pi}\,\text{sinc}\left(\frac{6t}{2\pi}\right) \leftrightarrow 2P_6(\omega)$$

$$\therefore \boxed{x(t) = \frac{6}{\pi}\,\text{sinc}\left(\frac{3t}{\pi}\right)}$$

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Odd function $\Rightarrow a_0 = 0,\ a_k = 0$

$$b_k = \frac{4}{T}\int_0^{T/2} x(t)\sin(k\omega_0 t)\,dt$$

$$T = 4 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$b_k = \int_0^2 2\sin\left(k\frac{\pi}{2}t\right)dt = 2\left[\frac{-1}{k\frac{\pi}{2}}\cos\left(k\frac{\pi}{2}t\right)\right]_0^2$$

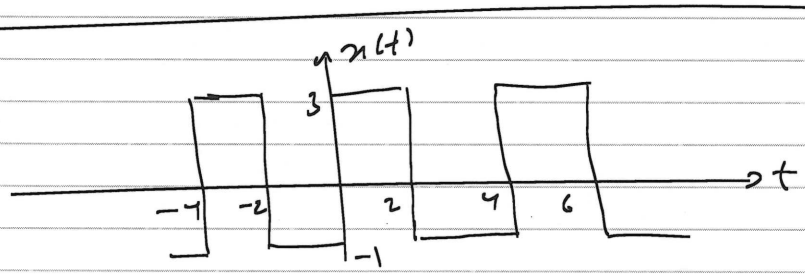
$$= \frac{-4}{k\pi}\left[\cos\left(k\frac{\pi}{2}t\right)\right]_0^2 = \frac{-4}{k\pi}\left[\cos(k\pi) - 1\right]$$

$$b_k = \frac{-4}{k\pi}\left[(-1)^k - 1\right]$$

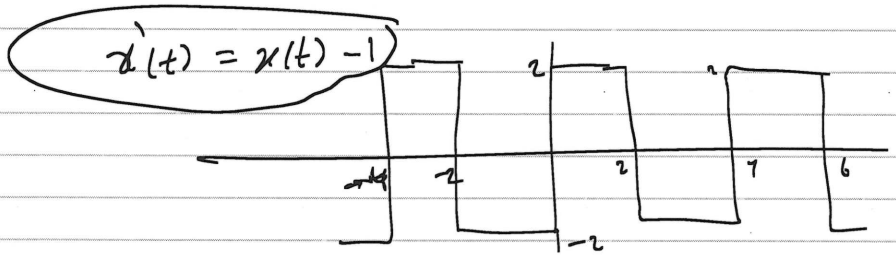
$$b_k = \begin{cases} \frac{8}{k\pi}, & \text{for } k_{odd} \\ 0, & \text{for } k_{even} \end{cases}$$

$$x(t) = \sum_{\substack{k=1 \\ k_{odd}}}^{N} \frac{8}{k\pi} \sin\left(k\frac{\pi}{2}t\right), \quad -\infty < t < \infty$$

(21)



$x'(t) = x(t) - 1$



$x'(t)$ is odd function

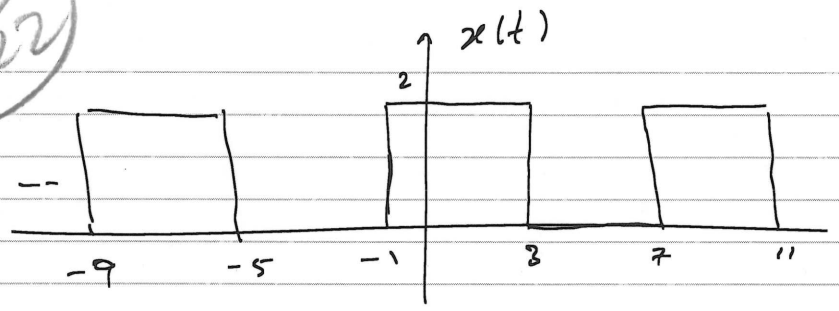
$a_0 = 0$, $a_k = 0$, $b_k = \begin{cases} \frac{8}{k\pi}, & k_{odd} \\ 0, & k_{even} \end{cases}$

$$x'(t) = \sum_{\substack{k=1 \\ k_{odd}}}^{N} \frac{8}{k\pi} \sin\left(k\frac{\pi}{2}t\right), \quad -\infty < t < \infty$$

$$x(t) = x'(t) + 1$$

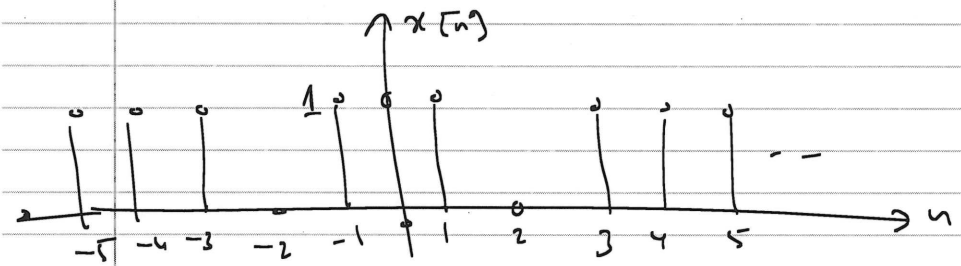
$$\therefore x(t) = 1 + \sum_{\substack{k=1 \\ k_{odd}}}^{N} \frac{8}{k\pi} \sin\left(k\frac{\pi}{2}t\right), \quad -\infty < t < \infty$$

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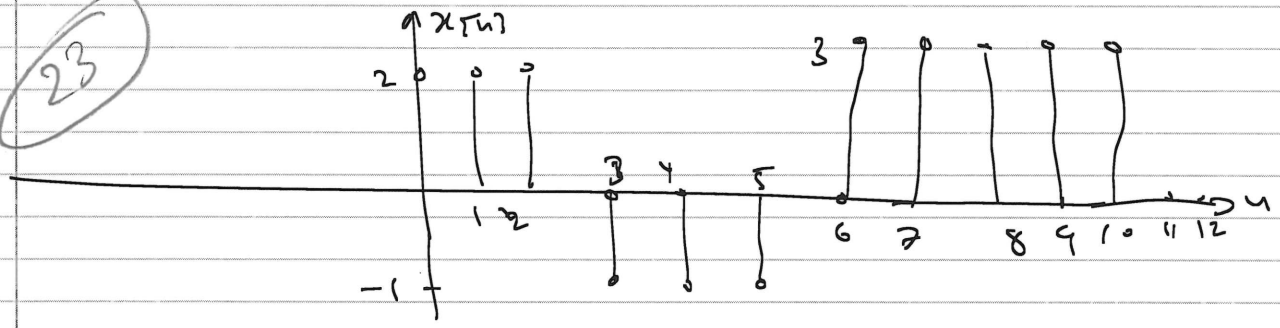


$\dot{x}(t) = x(t+1)$ ⇒ use Even function

or $\dot{x}(t) = x(t-1) - 1$ ⇒ use Odd function



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$$x[n] = 2P_2[n-1] - P_2[n-4] + 3P_4[n-8]$$

$$P_{2q}[n] \longleftrightarrow \frac{\sin\left[\left(q+\frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)}$$

$$X(\Omega) = 2\frac{\sin[1.5\Omega]}{\sin(\Omega/2)}e^{-j\Omega} - \frac{\sin[1.5\Omega]}{\sin(\Omega/2)}e^{-j4\Omega} + 3\frac{\sin[2.5\Omega]}{\sin(\Omega/2)}e^{-j8\Omega}$$

$$X(\omega) = \frac{2}{2+j\omega}$$

$$y(t) = x(t-2)$$

$$Y(\omega) = \frac{2}{2+j\omega} e^{-j2\omega}$$

$$y(t) = x(2t)$$

$$Y(\omega) = \frac{1}{2}\left(\frac{2}{2+j(\omega/2)}\right)$$

$$y(t) = \frac{dx(t)}{dt}$$

$$X(\omega) = j\omega \frac{dX(\omega)}{dt}$$